C. U. SHAH UNIVERSITY
Summer Examination-2022

Subject Name: Graph Theory

Subject Code: 4SC06GRT1   **Branch: B.Sc. (Mathematics)**

Semester: 6   **Date: 07/05/2022**   **Time: 02:30 To 05:30**   **Marks: 70**

Instructions:
    (1) Use of Programmable calculator & any other electronic instrument is prohibited.
    (2) Instructions written on main answer book are strictly to be obeyed.
    (3) Draw neat diagrams and figures (if necessary) at right places.
    (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**
    a) Define: Simple graph   01
    b) The degree of each vertex in complete graph K_n is   01
        (a) n -1 (b) n + 1 (c)2n (d) n
    c) Degree of pendant vertex is _____.   01
        (a) 3 (b) 2 (c) 1 (d) 0
    d) Define: Tree   01
    e) State Dirac's theorem   02
    f) Prove that a binary tree has always odd number of vertices.   02
    g) Define: Branches and chords of graph.   01
    h) Find rank and nullity for complete graph K_n.   02
    i) Define: Edge connectivity of a graph.   01
    j) Define: Maximal Hamiltonian graph.   01
   **k)** Define: Fundamental Circuit   01

Attempt any four questions from Q-2 to Q-8

Q-2   **Attempt all questions**   **(14)**
    a) State and prove first theorem of graph theory. Also prove that graph G **07**
        must have even number of odd vertices.
    b) Let $G = (V, E)$ be a graph where $V = \{a, b, c, d, e\}$ and   **07**
        $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and correspondence between elements of V
        and E are $e_1 = ab, e_2 = bc, e_3 = cc, e_4 = cd, \ e_5 = bd, \ e_6 = de, e_7 =$
        be then represent G as graphically and give the answer of following
        questions
        (i) Find isolated vertex of G
        (ii) Find pendent vertex of G
        (iii) Find even and odd vertices of G.
        (iv) Verify first theorem of graph theory.
   (v) Verify that number of odd vertices in graph is even.

Q-3   **Attempt all questions**   **(14)**
   a) State and prove Euler's theorem.   **06**
   b) Let $G = (V, E)$ be $k-$ regular graph, where k is an odd number then prove that number of edges in graph is multiple of k.   **04**
   c) Prove that number of edges in complete graph K_n is $\frac{n\,(n-1)}{2}$.   **04**

Q-4   **Attempt all questions**   **(14)**
   a) Define isomorphism of graphs. Show that the following graphs are isomorphic.   **05**
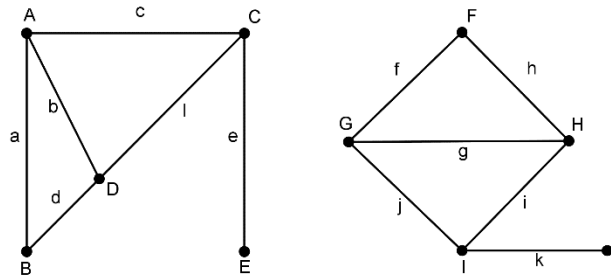
   b) Find distance between every pair of vertices of G and eccentricity of every vertex.   **05**
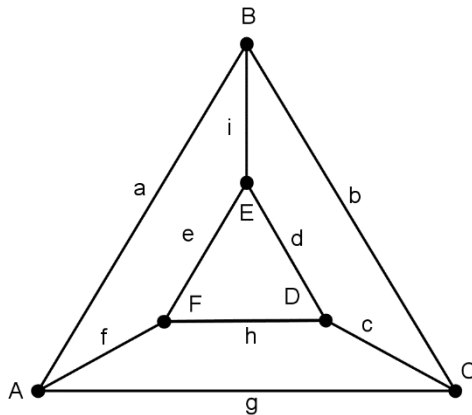
   c) Draw dodecahedron graph. Show that it is Hamiltonian graph.   **04**

Q-5   **Attempt all questions**   **(14)**
   a) Let G be a simple graph with n vertices and k components. Then prove that G can have at most $\frac{(n-k)(n-k+1)}{2}$ number of edges.   **05**
   b) Let G be a tree with n vertices. Then prove that G has $(n-1)$edges.   **05**
   c) Let G be acyclic graph with n vertices and k components then prove that G has $n - k$ edges.   **04**

Q-6   **Attempt all questions**   **(14)**
   a) Let n be an odd number, $n \geq 3$. Then prove that there are exactly $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuit in complete graph K_n.   **07**
   b) If the number of vertices is n in binary tree then prove that the number of pendant vertices is $\frac{n+1}{2}$.   **04**

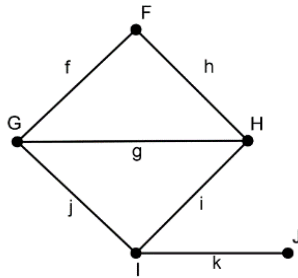**c)** Find all cut set of following graphs. **03**
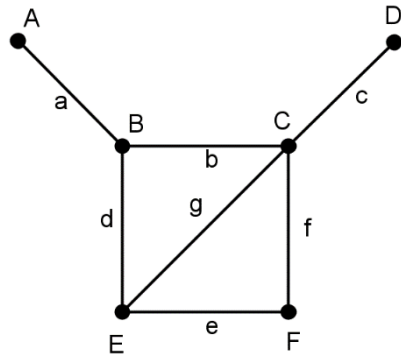
Q-7 **Attempt all questions** **(14)**
- **a)** State and prove necessary and sufficient condition for disconnected graph. **06**
- **b)** Prove that a graph G is tree if and only if it is minimal connected graph. **05**
- **c)** Define: Fusion graph. Find a fusion graph of the following graph by fusing the vertices F and H. **03**

Q-8 **Attempt all questions** **(14)**
- **a)** Without drawing graph check whether the graph corresponding to following adjacency matrix is connected or not. **07**

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

- **b)** Write adjacency and incidence matrices of the adjacent graph: **07**